Algebraic Topological Methods in Computer Science, II

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Contents

Efficient Algorithms for Computing the Betti Numbers of Semi-algebraic Sets

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In this talk, I will describe some new algorithms for computing the Betti numbers of semi-algebraic sets. More precisely, I will describe an algorithm for computing the first few Betti numbers of any given semi-algebraic set whose complexity is singly exponential in the dimension. Previously, only the zero-th Betti number was known to be computable in single exponential time. I will also describe polynomial time algorithms for computing certain Betti numbers of sets defined by quadratic inequalities.

Towards a model category for local po-spaces

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Topological spaces with a local partial order and locally increasing maps provide a framework for studying concurrency. We define a category of local po-spaces which satisfies some nice technical conditions and apply some formal constructions to obtain a category in which one can do homotopy theory.

Topology of Point Cloud Data

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We will discuss various methods for computing the homology of geometric objects from finite sets of points sampled from them (point cloud data). We will illustrate with an example how such computations can be useful in analyzing high dimensional data sets.

Stability of Persistence Diagrams
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Using topological persistence, one can associate with each (sufficiently regular) function over a topological space a multi-set of intervals, which encode the "topological attributes" of the function. These multi-sets of intervals can be depicted as multi-sets of points in the plane, which we call persistence diagrams. In this talk, I will show that the persistence diagram of a function behaves continuously when the function is perturbed in the C^0 sense. More precisely, the Frechet distance between the persistence diagrams of two functions is not greater than the maximum difference between the functions. As an application, I will show how the Betti numbers of a closed subset of R^n can be estimated from a set of point samples, under some sampling conditions.

A Barcode Shape Descriptor for Curve Point Cloud Data
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We present a complete computational pipeline for extracting a compact shape descriptor for curve point cloud data. Our shape descriptor, called a barcode, is based on a blend of techniques from differential geometry and algebraic topology. We also provide a metric over the space of barcodes, enabling fast comparison of PCDs for shape recognition and clustering. To demonstrate the feasibility of our approach, we have implemented our pipeline and provide experimental evidence in shape classification and parametrization.

Homotopy types of box complexes
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Matousek and Ziegler compared various topological lower bounds for the chromatic number. They proved that Lovasz's original bound can be restated as $\chi(G) \geq ind(B(G)) + 2$. Sarkaria's bound can be formulated as $\chi(G) \geq ind(B_0(G)) + 1$. It is known that these lower bounds are close to each other, namely the difference between them is at most 1. In this talk we will take a closer look to these lower bounds.

Elevation on a 2-manifold

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The elevation of a point on Earth is computed as the difference in height (distance from the center of mass) of the point and the sea level in the direction of the point. We use an extension of the algebraic concept of topological persistence to define a similar function on a 2-manifold that is not necessarily approximately spherical.

A Topological Approach to the game of "20 Questions"

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Since its introduction in 1925, Morse theory has been a fundamental tool in the study of the topology of smooth manifolds. We will show how a combinatorial version of Morse theory can be used to relate the complexity of some search problems - where the search is done by a sequence of yes/no questions - to the topological complexity of an associated topological space.

Reconfiguration and the geometry of cube complexes

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Numerous problems in theoretical computer science give rise to cubical complexes as configuration spaces. We demonstrate a general class of "local reconfigurable systems" for which the lack of [discrete] positive curvature is of great benefit in answering questions about optimization.

Algorithms for computing fundamental categories, and applications to the static analysis of concurrent programs

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This talk naturally follows Martin Raussen's talk on compressing the fundamental category of a set of concurrent processes. I will show some ways to define these compressions, their theoretical interest and some ways to compute these economic presentations, in simple but relevant (to concurrency) cases. I will show the practical interest of these apparently fairly theoretical objects, to validating in an efficient manner a concurrent program.

Quillen model categories applied to concurrency theory

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I will discuss two approaches to solving problems in concurrency theory, based on the model categories of homotopy theory. In the first we establish the existence of model category structure on the category of small 2-categories, which enables us to apply homotopy invariants to the classification of transition systems. The second approach involves developing a meaningful model category of locally partially-ordered spaces, which are the objects of directed topology.

Higher order automata, cubical sets, and some conjectures of Grothendieck

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Higher order automata are cubical complexes which give models for parallel processing algorithms via their associated homotopy types. Until recently, despite the fact that these objects have been known to topologists for more than fifty years, there was no combinatorial method for studying their homotopy types. A description of the combinatorial homotopy theory of cubical sets will be presented in this talk. The theory has all of the features that one knows from experience with simplicial sets, but the expected description of fibrations has a suprisingly subtle proof. This proof was given by Denis-Charles Cisinski, and involves the most subtle aspects of his work on Grothendieck's conjectures for models of homotopy types.

Convex hulls, simplicial homology, and suitable software

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It is shown how the standard convex hull problem can be reduced to a sequence of simplicial homology computations. Additionally, we report on a partial implementation of these methods in the software system polymake.

Particle spaces and associated mapping spaces

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To a given smooth manifold and a collection of ”particles” on it, topologized according to the way one wishes them to interact, it is possible to associate a function space, with source the manifold, and of which topological properties are identical to the particle space one starts with. Topological applications of this construction will be given.

Statistical Inverse Problems on Riemannian Manifolds

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This talk extends statistical inverse problems to compact Riemannian manifolds. The approach is to use aspects of spectral geometry associated with the Laplace-Beltrami operator on compact Riemannian manifolds. Although the vast majority of inverse problems usually take place in Euclidean space, certain physical problems can be genuinely non-Euclidean. Various applications to, bioinformatics, chemistry, imaging, and quantum computing, will be addressed.

Complexity and Tractability issues in Topological aspects of 3-D Computational Electromagnetics
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In this talk we will briefly review decidability and complexity issues in manifold theory as a function of dimension. This sets the stage for examining complexity issues in dimension three. We then show how these issues in low dimensional topology manifest themselves in the design of software tools for the analysis of three dimensional electromagnetic fields. In this way, deep computational issues in algebraic topology are related to everyday problems in engineering electromagnetics.

Topological obstructions to graph colorings
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For any two graphs G and H Lovász has defined a cell complex $Hom(G, H)$ having in mind the general program that the algebraic invariants of these complexes should provide obstructions to graph colorings. Here we announce the proof of a conjecture of Lovász concerning these complexes with G a cycle of odd length.

More specifically, we show that

If $Hom(C_{2r+1}, G)$ is k-connected, then $\chi(G) \geq k + 4$.

Our actual statement is somewhat sharper, as we find obstructions already in the non-vanishing of powers of certain Stiefel-Whitney characteristic classes.

Finite dynamical systems: a mathematical foundation for simulation science
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Large-scale computer simulations are playing an increasingly important role in the understanding of complex social, technological, and biological systems, such as road traffic networks, social networks involved in the spread of infectious diseases, and the immune system. Such systems can be described using interaction-based, discrete models with a finite number of possible states. This type of model captures important spatial features, for instance. The complexity of these simulations highlights two important issues: software verification and analysis of simulation output. It is desirable to have available a mathematical specification of simulations that can provide a rigorous basis for both.

In this talk we describe such a mathematical framework and some recent applications to the problem of understanding the relationship between simulation structure and the resulting dynamics, that is, the topology of its state space. To be precise, we describe a category of finite dynamical systems and derive some of its properties, together with a functor to the category of state spaces. We also describe several families of finite dynamical systems whose structure provides information about the structure of the state space.

Graph Coloring Manifolds

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In the topological approach to graph coloring problems, initiated by Lovász, lower bounds on the chromatic number $\chi(G)$ of a graph G are obtained by associating certain simplicial or cell complexes to G and then exploiting topological invariants of the resulting spaces. Computer calculation of these invariants is a standard first step to analyze and understand explicit examples, often leading to more general conjectures or theorems.

Recently, Babson and Kozlov proved Lovász' conjecture that if for a graph H the cell complex $\mathrm{Hom}(C_{2r+1}, H)$, introduced by Lovász, is k-connected for some $r \geq 1$, then $\chi(H) \geq k+4$.

In this talk, we will show that, in fact, $\mathrm{Hom}(C_5, K_{n+2})$ is an $(n-1)$-sphere bundle over the n-sphere. Moreover, we will characterize the graphs G for which $\mathrm{Hom}(G, K_n)$ are manifolds, and, in this way, obtain a new and rich class of *graph coloring manifolds.*

Homotopical properties for concurrent systems

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In this talk, I will consider the models of concurrent systems in rewriting logic. I will present a homotopical invariant for classification of such systems by using crossed resolutions of algebraic theories.

Computer Intrusion Detection Using Features from Graph Theory and Algebraic Topology

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This talk will describe the problem of profiling the users of a network for the purpose of intrusion detection. We look at 3 graphs associated with each session on a Windows NT machine. A number of features are extracted from these graphs and fed to a classifier which uses "random forest" techniques for distinguishing between the graphs associated with one user and those associated with another. The features can be taken from the properties of the graphs themselves as well as from a variety of simplicial complexes which can be built using the information contained in the graphs.

The talk will present some initial results as well as a number of ideas we plan to pursue in the near future.

A categorical approach for parallel Delaunay mesh generation

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One of the challenges for stable parallel Delaunay mesh generations is concurrency. Partial order is a tool for maintaining stability in the context of concurrency. On the other hand, partially ordered sets are one of the basic examples of categories. We will develop a theoretical framework using category theory to study the stability of parallel 2-dimensional Delaunay triangulations based on the Bowyer-Watson kernel. Ultimately the triangulation becomes the homotopy colimit of certain functors.

Fundamental categories with a view to concurrency

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In geometric models for concurrency, a computation corresponds to a path with a preferred direction, and a homotopy respecting directions gives rise to equivalent computations. This motivates the study of the fundamental category of a state space, of methods compressing its size without loosing information and of tools for its calculation.

Harmonic methods in computational topology

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Suppose that a topological subspace of Euclidean space is represented by a finite point sample. How does one recover topological invariants of the space using constructions which depend only on the sample points? A standard paradigm is to build a nested family of simplicial complexes and then calculate persistent homology groups using exact arithmetic in a finite field. I will discuss an alternative paradigm using floating point arithmetic with discrete Laplacian operators.

Vector Field Design on Surfaces

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Vector field design on surfaces is necessary for many graphics applications: example-based texture synthesis, non-photorealistic rendering, and fluid simulation. For these applications, singularities contained in the input vector field often cause visual artifacts. In this paper, we present a vector field design system that allows a user to create a wide variety of vector fields with control over vector field topology, such as the number and location the singularities. Our system combines basis vector fields to make an initial vector field that meets the user specifications.

The initial vector field often contains unwanted singularities. Such singularities cannot always be eliminated, due to the Poincaré-Hopf index theorem. To reduce the visual artifacts caused by these singularities, our system allows a user to move a singularity to a more favorable location or to cancel a pair of singularities. These operations provide topological guarantees for the vector field in that they only affect the user-specified singularities. Other editing operations are also provided so that the user may change the topological and geometric characteristics of the vector field.

To create continuous vector fields on curved surfaces represented as meshes, we make use of the ideas of exponential map and parallel transport to interpolate vector values defined at the vertices of the mesh. We also use exponential map and parallel transport to create basis vector fields on surfaces that meet the user specifications. These techniques allow our vector field design system to work for both planar domains and curved surfaces.

We demonstrate our vector field design system for several applications: example-based texture synthesis, painterly rendering of images, and pencil sketch illustrations of smooth surfaces.

Persistence Barcodes for Shapes

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I will talk about a study of shape description and classification via the application of persistent homology to two tangential constructions on geometric objects. Our techniques combine the differentiating power of geometry with the classifying power of topology. The homology of our first construction, the tangent complex, can distinguish between topologically identical shapes with different "sharp" features, such as corners. To capture "soft" curvature-dependent features, we define a second complex, the filtered tangent complex, obtained by parametrizing a family of increasing subcomplexes of the tangent complex. Applying persistent homology, we obtain a shape descriptor, called a barcode, that is a finite union of intervals. We define a metric over the space of such intervals, arriving at a continuous invariant that reflects the geometric properties of shapes. We illustrate the power of our methods through a number of detailed studies of parametrized families of mathematical shapes.